TIME-VARYING FILTER BANKS WITH VARIABLE SYSTEM DELAY

Gerald Schuller

University of Hannover
Institut fuer Theoretische Nachrichtentechnik und Informationsverarbeitung
30167 Hannover, Germany
email schuller@tnt.uni-hannover.de

ABSTRACT

A new filter structure and design method for time-varying cosine modulated FIR filter banks with critical sampling, perfect reconstruction, and an efficient implementation is presented. The proposed filter banks have an arbitrary system delay which can be chosen in the design process and is independent of the arbitrary filter length, hence making a low system delay possible. The time variation includes changing the number of bands and/or filters during signal processing while maintaining critical sampling and perfect reconstruction. The transition windows can be overlapping, which improves the frequency responses. It is based on a factorization of the polyphase matrices into a cascade of 2 types of simple matrices.

1. INTRODUCTION

The system considered here is an N band (N not necessarily even) cosine modulated FIR filter bank with critical downsampling and perfect reconstruction (PR). If a support preservative PR filter bank is time varying the filters and sometimes also the number of subbands are changed while processing a signal in such a way that the input signal can still be reconstructed by the synthesis filter bank and the total number of subband samples does not exceed the number of input samples [1, 2, 3, 4, 5]. This has the advantage that the filter bank can be adapted e.g. to changing signal statistics in coding applications even during operation, which leads to a higher coding gain and reduced coding artifacts.

Stationary low-delay (or bi-orthogonal) cosine-modulated filter banks have first been introduced in [7, 8, 10], later a different approach was used in [6]. This makes filter banks with non-symmetric filters and a lower than usual delay possible, which is important for applications like speech and audio coding. However, the problem of time-varying bi-orthogonal filter banks with changing numbers of subbands has not been addressed in the literature before and will be addressed in the following using the factorization or cascade described in [12], which is computationally efficient and PR even if low precision arithmetic is used. Its system delay does not need to be an integer multiple of N, it can be specified in terms of the input sampling rate, so that it can be matched more closely to some requirements.

The analysis and synthesis impulse responses, resp., are given by:

$$h_k(n) = h(n) \cdot \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5+n_0)\right) \quad (1)$$

$$g_k(n) = h'(n) \cdot \frac{2}{N} \cdot \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5-N+n_0')\right) \quad (2)$$

$k = 0, \ldots, N-1$; $n = 0, \ldots, LN-1$;$-N \leq n_0, n_0' \leq N$; where k is the frequency or band index,and $h(n)$ and $h'(n)$ are the analysis and synthesis baseband prototype filters, respectively. Their lengths LN can include leading or trailing zeros. The scaling factor of $2/N$ simplifies the following notation. The downsampled analysis filter output is $y_k(m)$, where m is the index at the lower sampling rate. The filter bank is PR if the synthesis output signal $\hat{x}(n)$ is identical to the analysis input signal delayed by n_d samples, $\hat{x}(n) = x(n-n_d)$, where n_d is the system delay. Orthogonal or unitary filter banks have a standard system delay which equals the length of its filters minus one sample.

1.1. Definitions

Boldface letters denote matrices or vectors, capital letters z-transforms. ":=" means "defined as". A polynomial matrix $\mathbf{F}(z)$ is causal if it contains no positive powers of z. $\mathbf{I}$ and $\mathbf{J}$ denote the $N \times N$ identity and counter identity matrix resp., $\mathbf{diag}$ is an $N \times N$ diagonal coefficient matrix and $\mathbf{S}(z)$ is a shift matrix.

$$\mathbf{J} := \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{diag}(x_0, \ldots, x_{N-1}) := \begin{bmatrix} x_0 & 0 & \cdots & 0 \\ 0 & x_1 & \cdots & 0 \\ & & \ddots & \vdots \\ 0 & 0 & \cdots & x_{N-1} \end{bmatrix}$$

$$\mathbf{S}(z) := \begin{bmatrix} 0 & 0 & \cdots & 0 & z \\ 1 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$[\mathbf{F}]_{n,k}$ denotes the element at the n'th row and k'th column of the matrix $\mathbf{F}$, $\prod_{j=1}^{\nu} \mathbf{L}_j := \mathbf{L}_1 \cdot \ldots \cdot \mathbf{L}_\nu$ (the ordering is important).

2. THE POLYPHASE NOTATION

The basis of the theory is the polyphase formulation. The effect of downsampling and upsampling in the analysis and synthesis filter bank, respectively, can be viewed as processing the signal in blocks of length N. The analysis input is represented by an N-dimensional row vector $\mathbf{x}(m)$ composed of sequences of the downsampled $x(n)$,

$$\begin{aligned} \mathbf{x}(m) &:= [x(mN), \ldots, x(mN+N-1)] \\ &:= [x_0(m), \ldots, x_{N-1}(m)] \end{aligned}$$

The z-transform of $\mathbf{x}(m)$ is the row vector

$$\mathbf{X}(z) = [X_0(z), \ldots, X_{N-1}(z)]$$

The analysis output is represented by the z-domain row vector $\mathbf{Y}(z) = [Y_0(z), \ldots, Y_{N-1}(z)]$ and the synthesis output by $\hat{\mathbf{X}}(z)$. The analysis type-2 polyphase matrix is $\mathbf{P_a}$, its elements are

$$[\mathbf{P_a}(z)]_{n,k} := \sum_{m=0}^{L-1} h_k(mN + N - 1 - n)z^{-m}$$

$n, k = 0, \ldots, N-1$. The synthesis type-1 polyphase matrix is

$$[\mathbf{P_s}(z)]_{k,n} := \sum_{m=0}^{L-1} g_k(mN + n)z^{-m}$$

The analysis filtering and downsampling and the synthesis filtering and upsampling operation can then be written as

$$\mathbf{Y}(z) = \mathbf{X}(z) \cdot \mathbf{P_a}(z) \ , \quad \hat{\mathbf{X}}(z) = \mathbf{Y}(z) \cdot \mathbf{P_s}(z)$$

The filter bank is PR if $\mathbf{P_a}(z) \cdot \mathbf{P_s}(z) = z^{-d} \cdot \mathbf{S}^{n_t}(z)$, for some integer d and $n_t < N \cdot d$ (for causality). This is a delay by d size N blocks and an advance by n_t samples at the input sampling rate. With the blocking delay of length $N-1$, which results from forming input blocks $\mathbf{X}(z)$ of length N before processing them, the system delay is $n_d = N - 1 + d \cdot N - n_t$, where n_t can be used for the "fine tuning" of the system delay.

3. THE NEW FILTER STRUCTURE

The time-varying filter banks that will be introduced are based on a new formulation for modulated FIR filter banks [12], which is briefly described next. The key for the new filter bank design is the factorization of the polyphase matrices in a product of sparse "filter matrices" with polynomial elements on their diagonal and anti-diagonal, transform matrices $\mathbf{T_a}, \mathbf{T_s}$, and the shift matrix $\mathbf{S}(z)$. They all can be implemented efficiently. The filter matrices are such that analysis and synthesis both have FIR filters, according to (1) and (2). Since there are 2 independent variables in the design process, the system delay and the filter length, 2 types of matrices are needed. The first are zero-delay matrices, who have the characterizing property that their inverse is causal. Including them in the product increases the length of the filters by N, but not the system delay. The second type of key matrices are maximum-delay matrices, who have an inverse which needs a multiplication with z^{-2} to make it causal. Their use also increases the filter length by N, but the system delay by $2N$. They can be seen as each, the matrix and its inverse, delaying the signal by one step at the lower sampling rate. Both types have a sparse structure, which leads to the efficient implementation, and are given below.

Zero-Delay Matrices– They increase the filter length but not the system delay.

$$\mathbf{E}_i(z) := \mathbf{J} + z^{-1} \cdot \mathbf{diag}(\underbrace{0, \ldots, 0}_{\lceil N/2 \rceil}, e^i_{\lceil N/2 \rceil}, \ldots, e^i_{N-1})$$

$$\mathbf{G}_i(z) := \mathbf{J} + z^{-1} \cdot \mathbf{diag}(g^i_0, \ldots, g^i_{\lfloor N/2 \rfloor - 1}, 0, \ldots, 0)$$

where e^i_j, g^i_j are matrix coefficients, and i denotes different sets of coefficients ($i > 0$). Observe that their inverse is causal, so that no multiplication with a delay is necessary.

$$\mathbf{E}^{-1}_i(z) = \mathbf{J} - z^{-1} \cdot \mathbf{diag}(e^i_{N-1}, \ldots, e^i_{\lceil N/2 \rceil}, 0, \ldots, 0)$$

$$\mathbf{G}^{-1}_i(z) = \mathbf{J} - z^{-1} \cdot \mathbf{diag}(0, \ldots, 0, g^i_{\lfloor N/2 \rfloor - 1}, \ldots, g^i_0)$$

Maximum-Delay Matrices– They also increase the filter length, but especially the system delay.

$$\mathbf{A}_i(z) := z^{-1} \cdot \mathbf{J} + \mathbf{diag}(0, \ldots, 0, a^i_{\lceil N/2 \rceil}, \ldots, a^i_{N-1})$$

$$\mathbf{B}_i(z) := z^{-1} \cdot \mathbf{J} + \mathbf{diag}(b^i_0, \ldots, b^i_{\lfloor N/2 \rfloor - 1}, 0, \ldots, 0)$$

The matrix $\mathbf{B}_0(z)$ uses also coefficients on the anti-diagonal, because there must be one matrix in the cascade which is not "normalized",

$$\mathbf{B}_0(z) := [z^{-1} \cdot \mathbf{diag}(b^0_N, \ldots, b^0_{2N-1}) \cdot \mathbf{J} +$$
$$+ \mathbf{diag}(b^0_0, \ldots, b^0_{\lfloor N/2 \rfloor - 1}, 0, \ldots, 0)] \cdot \mathbf{J}^{i_a}$$

where the exponent $i_a = 0$ if $n_0 > 0$, else $i_a = 1$. The $\mathbf{J}^{i_a}$ takes care of the correct form of the resulting analysis polyphase matrix in order to obtain filters of the form of (1). Their inverse need a multiplication with z^{-2} to obtain a causal matrix.

$$\mathbf{A}^{-1}_i(z) \cdot z^{-2} =$$
$$z^{-1} \cdot \mathbf{J} - \mathbf{diag}(a^i_{N-1}, \ldots, a^i_{\lceil N/2 \rceil}, 0, \ldots, 0),$$
$$\mathbf{B}^{-1}_i(z) \cdot z^{-2} =$$
$$z^{-1} \cdot \mathbf{J} - \mathbf{diag}(0, \ldots, 0, b^i_{\lfloor N/2 \rfloor - 1}, \ldots, b^i_0),$$
$$\mathbf{B}^{-1}_0(z) \cdot z^{-2} = \mathbf{J}^{i_a}[z^{-1} \cdot \mathbf{J} \cdot \mathbf{diag}(\hat{b}^0_N, \ldots, \hat{b}^0_{2N-1}) +$$
$$+ \mathbf{diag}(0, \ldots, 0, \hat{b}^0_{\lceil N/2 \rceil}, \ldots, \hat{b}^0_{N-1})]$$

with $\hat{b}^0_{N/2+j} = -b^0_j / (b^0_{N+j} b^0_{2N-1-j})$, $j = 0, \ldots, N/2 - 1$, and $\hat{b}^0_{N+j} = 1/b^0_{2N-1-j}$, $j = 0, \ldots, N-1$.

Furthermore an (analysis) transform matrix $\mathbf{T_a}$ is needed, which in this case will be a DCT 4 defined as $[\mathbf{T_a}]_{n,k} := \cos(\frac{\pi}{N}(k + 0.5)(n + 0.5)) \ , \ \ 0 \leq n, k < N$. It can be implemented e.g. as a fast DCT. The analysis polyphase matrix for filters as in (1) can now be written as

$$\mathbf{P_a}(z) = \mathbf{S}^{n_a}(z) \cdot \mathbf{B}_0(z) \cdot \prod_{i=1}^{\mu-1} \mathbf{H}_i(z) \cdot \prod_{j=1}^{\nu} \mathbf{L}_j(z) \cdot \mathbf{T_a}$$

where $\nu \geq 0$ and $\mu \geq 1$ are the number of zero-delay matrices and maximum-delay matrices resp., and $n_a = n_0$ if $n_0 > 0$, else $n_a = n_0 + N$. $\mathbf{H}_i$ and $\mathbf{L}_i$ are defined as $\mathbf{H}_i(z) := \mathbf{B}_i(z)$ if $n_0 > 0$, else $\mathbf{H}_i(z) := \mathbf{A}_i(z)$. $\mathbf{L}_i(z) := \mathbf{E}_i(z)$ if $n_0 > 0$ and μ is even or $n_0 \leq 0$ and μ is odd, else $\mathbf{L}_i(z) := \mathbf{G}_i(z)$. The coefficients of $\mathbf{B}_0(z)$ which lead to coefficients of $\mathbf{S}^{n_a}(z) \cdot \mathbf{B}_0(z)$ with positive powers have to be set to zero in order to obtain causal filters. $n_s \leq n_a$ ensures the same for the synthesis. The synthesis polyphase matrix for perfect reconstruction is

$$\mathbf{P_s}(z) = \mathbf{P_a}^{-1}(z) z^{-d} \cdot \mathbf{S}^{n_a + n_s}(z) =$$

$$\mathbf{T_s} \cdot \prod_{j=\nu}^{1} \mathbf{L}^{-1}_j(z) \cdot \prod_{i=\mu-1}^{1} \left(\mathbf{H}^{-1}_i(z) \cdot z^{-2}\right) \cdot \mathbf{B}^{-1}_0(z) \cdot z^{-2} \cdot \mathbf{S}^{n_s}(z)$$

with a synthesis transform matrix $\mathbf{T_s}$ such that $\mathbf{T_a} \cdot \mathbf{T_s} = \mathbf{I}$, and where $d = 2\mu$. The resulting synthesis impulse responses are as (2) with $n'_0 = n_s$ if $n_0 > 0$, else $n'_0 = n_s - N$. The system delay is

$$n_d = \mu \cdot 2N + N - 1 - n_a - n_s$$

As can be seen the minimum possible delay is the blocking delay of $N - 1$ samples. It is obtained with $\mu = 1$, $n_a = n_s = N$. The length of the non-zero part of the analysis and synthesis filters is $(\mu + \nu)N + \lfloor N/2 \rfloor - \max(\lfloor N/2 \rfloor, n_a)$ for $\nu > 0$ and $\mu N + N - \max(\lfloor N/2 \rfloor, n_a)$ for $\nu = 0$, where max(.,.) is the maximum of the two values. Filter banks with the traditional standard delay result e.g. if $\mu = \nu$ and $n_a = n_s = N/2$ is chosen, and with a low delay if $\mu < \nu$. This is a complete factorization for all cosine modulated filter banks with contiguous impulse responses, as shown in [12]. There it is also shown that if the transform is a DCT 4 and the coefficients of $\mathbf{B}_0(z)$ are such that $b^0_{N+i} = s/b^0_{2N-1-i}$ for $i = 0, \dots, N-1$, where s is either 1 or -1 for all i, then the baseband impulse responses for the analysis and synthesis are identical, except for the sign. This can be used for the filter bank design. The coefficients of the filter matrices determine the frequency responses of the filter bank, they can be obtained e.g. with the optimization method described in [9, 10, 11].

4. TIME VARIANCE

To make this filter bank time-varying its polyphase matrices and their components have to be time-varying. To express this time dependency the parameter m, denoting the time instance at the lower sampling rate, is introduced, $\mathbf{P_a}(z)$ becomes $\mathbf{P_a}(z,m)$, $\mathbf{A}_i(z)$ becomes $\mathbf{A}_i(z,m)$,

$$\mathbf{A}_i(z,m) := z^{-1}\cdot\mathbf{J}+\mathbf{diag}(0,\dots,0,a^i_{\lceil N/2\rceil}(m),\dots,a^i_{N-1}(m))$$

$\mathbf{E}_i(z)$ becomes $\mathbf{E}_i(z,m)$, and so forth. This additional parameter requires a computation which is different from the time invariant case. Observe that if a signal first passes a time-varying system or matrix $\mathbf{F}(z,m)$ and then a delay z^{-1}, the output is the same as if the signal is first delayed and then passes the system or matrix at the state of the previous time step. This is an important observation for the treatment of time-varying systems in the z-domain, and can be written as

$$\mathbf{F}(z,m)\cdot z^{-1} = z^{-1}\cdot\mathbf{F}(z,m-1)$$

This basic rule makes the computation of the inverses of the time-varying matrices and hence the computation of the time-varying synthesis polyphase matrix for perfect reconstruction possible. The inverses of the filter matrices are now

$$\mathbf{E}_i^{-1}(z,m) =$$

$$\mathbf{J} - z^{-1}\cdot\mathbf{diag}(e^i_{N-1}(m),\dots,e^i_{\lceil N/2\rceil}(m),0,\dots,0)$$

$$\mathbf{G}_i^{-1}(z,m) =$$

$$\mathbf{J} - z^{-1}\cdot\mathbf{diag}(0,\dots,0,g^i_{\lfloor N/2\rfloor-1}(m),\dots,g^i_0(m))$$

$$\mathbf{A}_i^{-1}(z,m)\cdot z^{-2} = z^{-1}\cdot\mathbf{J}$$

$$-\mathbf{diag}(a^i_{N-1}(m-1),\dots,a^i_{\lceil N/2\rceil}(m-1),0,\dots,0),$$

$$\mathbf{B}_i^{-1}(z,m)\cdot z^{-2} = z^{-1}\cdot\mathbf{J}$$

$$-\mathbf{diag}(0,\dots,0,b^i_{\lfloor N/2\rfloor-1}(m-1),\dots,b^i_0(m-1)),$$

$$\mathbf{B}_0^{-1}(z,m)\cdot z^{-2} =$$

$$\mathbf{J}^{i_a}[z^{-1}\cdot\mathbf{diag}(\hat{b}^0_N(m),\dots,\hat{b}^0_{2N-1}(m))\cdot\mathbf{J}+$$

$$+\mathbf{diag}(0,\dots,0,\hat{b}^0_{\lceil N/2\rceil}(m),\dots,\hat{b}^0_{N-1}(m))]$$

with

$$\hat{b}^0_{N-1-j}(m) = \frac{-b^0_j(m-1)}{b^0_{N+j}(m-1)b^0_{2N-1-j}(m)}$$

for $j = 0,\dots,N/2-1$, and $\hat{b}^0_{N+j}(m) = 1/b^0_{2N-1-j}(m-1)$ for $j = 0,\dots,N-1$

A delay between a time-varying matrix and its inverse results in a time shift in the inverse, because the input to the inverse matrix is now a delayed version of the matrix. This can be seen in

$$\mathbf{F}(z,m)\cdot z^{-d}\cdot\mathbf{F}^{-1}(z,m-d) = \tag{3}$$

$$= z^{-d}\cdot\mathbf{F}(z,m-d)\cdot\mathbf{F}^{-1}(z,m-d) = z^{-d}\cdot\mathbf{I}$$

The time-varying analysis filter bank can now be expressed as

$$\mathbf{P_a}(z,m) =$$

$$= \mathbf{S}^{n_a}(z)\cdot\mathbf{B}_0(z,m)\cdot\prod_{i=1}^{\mu-1}\mathbf{H}_i(z,m)\cdot\prod_{j=1}^{\nu}\mathbf{L}_j(z,m)\cdot\mathbf{T_a}(m)$$

To obtain perfect reconstruction from this analysis polyphase matrix the synthesis polyphase matrix has to have the form

$$\mathbf{P_s}(z,m) = \mathbf{P_a}^{-1}(z,m)z^{-d} =$$

$$\mathbf{T_s}(m)\cdot\prod_{j=\nu}^{1}\mathbf{L}_j^{-1}(z,m)\cdot\prod_{i=\mu-1}^{1}\left(\mathbf{H}_i^{-1}(z,m-2(\mu-1-i))z^{-2}\right)\cdot$$

$$\cdot\left(\mathbf{B}_0^{-1}(z,m-2(\mu-1))z^{-2}\cdot\mathbf{S}^{n_s}(z)\right) \tag{4}$$

where $\mathbf{T_a}(m)\cdot\mathbf{T_s}(m) = \mathbf{I}$. Observe the time indices in (4). The signal can be viewed as passing the matrices from left to right. Since the $\mathbf{L}^{-1}$ matrices don't introduce any additional system delay their time index is still m. The $\mathbf{H}^{-1}$ matrices are associated with an additional delay (z^{-2}), and for this reason the time index after each of these delays has to be lowered by 2 in order to obtain perfect reconstruction, as seen in (3). n_a, μ, and ν do not change, they are not time dependent so that the total delay is constant, as needed for perfect reconstruction. Note that this is a very general approach that accommodates many different ways of switching the analysis and synthesis filters.

4.1. Switching the Number of Bands

To use the above formulation for filter banks with different numbers of bands, they must use signal vectors $\mathbf{x}$ and matrices of the same dimension. Assume any 2 different numbers of bands N_1, N_2, and assume $N_1 > N_2$. They can both be described with signal vectors of size N_1. First consider the time invariant case and define

$$\mathbf{X}'(z) := \mathbf{X}(z)\cdot\mathbf{S}^{n_a}(z) \tag{5}$$

which is the input vector for the filter matrices. $\mathbf{X}'$ and $\mathbf{X}$ are of length N_1 and $\mathbf{S}(z)$ is of size $N_1\times N_1$. The case with N_1 bands is obvious. For the second case, N_2 bands, $n_a = n'_a\cdot N_1/N_2$ where n'_a is from the size N_2 case. n_a and n'_a must be such that both are integers. Then $N_1 - N_2$ zeros are placed in $\mathbf{X}(z)$ such that $\mathbf{X}'(z)$ has the zeros around the center, $\mathbf{X}'(z) =$

$$[X'_0(z),\dots,X'_{N_2/2-1}(z),\underbrace{0,\dots,0}_{N_1-N_2},X'_{N_2/2}(z),\dots,X'_{N_2-1}(z)]$$

e.g. by computing $\mathbf{X}(z) = \mathbf{X}'(z)\cdot\mathbf{S}^{-n_a}(z)$ (odd numbers of bands would lead to one non-zero value also in the center). The filter matrices are now $N_1\times N_1$ matrices of the form

$$\mathbf{A}_i(z) = z^{-1}\cdot\mathbf{J}+\mathbf{diag}(\underbrace{0,\dots,0}_{N_1-\lfloor N_2/2\rfloor},a^i_{\lceil N_2/2\rceil},\dots,a^i_{N_2-1})$$

$$\mathbf{B}_0(z) = [z^{-1}\cdot\mathbf{diag}(b^0_{N_2},\dots,b^0_{N_2+\lfloor N_2/2\rfloor-1},$$

$$1,\dots,1,b^0_{N_2+\lceil N_2/2\rceil},\dots,b^0_{2N_2-1})\cdot\mathbf{J}+$$

$$+\mathbf{diag}(b^0_0,\dots,b^0_{\lfloor N_2/2\rfloor-1},0,\dots,0)]\cdot\mathbf{J}^{i_a}$$

and so forth. Odd numbers of bands would lead to one coefficient for z^{-1} in the center of $\mathbf{B}_0(z)$ which can be unequal to 1. Because the filter matrices have non-zero coefficients only on the diagonal and anti-diagonal, the zeros appear also at the transform. Since they are known, they don't need to be processed further and can be omitted for the computation of the transform, so that an analysis transform of size $N_2 \times N_1$ can be used which results from splitting the $N_2 \times N_2$ transform matrix into an upper and lower half ($\mathbf{T}_u$ and $\mathbf{T}_d$ resp.) and inserting $N_1 - N_2$ rows of zeros in the middle. The synthesis transform is an $N_2 \times N_1$ matrix which results from splitting the $N_2 \times N_2$ inverse transform matrix into a left and right half ($\mathbf{T}_l$ and $\mathbf{T}_r$ resp.) and inserting $N_1 - N_2$ columns of zeros in the middle.

$$\mathbf{T_a} = \begin{bmatrix} \mathbf{T}_u \\ \hline \mathbf{0} \\ \hline \mathbf{T}_d \end{bmatrix}, \quad \mathbf{T_s} = [\ \mathbf{T}_l \mid \mathbf{0} \mid \mathbf{T}_r\ ]$$

Their product is the $N_1 \times N_1$ matrix

$$\mathbf{T_a} \cdot \mathbf{T_s} = \left[\begin{array}{c|c|c} \mathbf{I} & & \mathbf{0} \\ & \mathbf{0} & \\ \mathbf{0} & & \mathbf{I} \end{array} \right]$$

In the case of an odd number of bands the analysis transform would also have one nonzero row in the middle, and the synthesis transform would have one nonzero column in the middle, so that their product would have a 1 in its center. The so defined length N_1 signal vectors and size $N_1 \times N_1$ filter matrices now represent a filter bank with N_2 bands. Note that the computational complexity is the same as for signal vectors and matrices of size N_2, since only operations with non-zero coefficients need to be computed.

This formulation can now be used for switching between different numbers of bands. The transition between N_1 and N_2 mode can be done in different ways, e.g. by switching $\mathbf{x}'(m)$ directly between the forms of the time invariant cases and computing the resulting transition for $\mathbf{x}(m)$, which results in a direct switch between the 2 numbers of bands with an intermediate blocksize of the processed input during the transition, if $|n_a| > N_2/2$, as can be seen by applying (5). Or by switching $\mathbf{x}(m)$ directly between the two forms. which results in a direct switch in the blocksize of the processed input signal and an intermediate number of bands. The transform matrix has to be switched such that it has the suitable entries of zeros to maintain PR and critical sampling.

Assume the switch begins at time m_0 with the switch of $\mathbf{x}'(m)$ or $\mathbf{x}(m)$ to the other mode. The maximum delay matrices can be switched to the other mode while the other mode signal passes them, one step after the other (at the lower sampling rate), since each delays the signal by one step, and then the minimum delay matrices and the transform are switched at once, since they don't delay the signal. This means $\mathbf{B}_0(z,m)$ is switched to the other mode at time $m = m_0 + 1$, $\mathbf{H}_i(z,m)$ at time $m = m_0 + 1 + i$, $\mathbf{L}_i(z,m)$ and $\mathbf{T_a}(m)$ at time $m = m_0 + \mu$. Switching this way ensures critical downsampling since the other mode signal is delayed by the same number of steps independent of the switching direction, and perfect reconstruction is maintained by using the synthesis as in (4). A change of the filter matrices just before and after the mode switch can be used to obtain different transition filters. The minimum number of "windows" affected by a switch is the number of overlapping windows, which is L. The time from the beginning of the switch until the number of bands is in the new mode at the analysis is the time the mode change needs to pass the shift- and maximum-delay matrices.

If $N_2 = 0$ then this scheme can be used for designing filters for boundary regions of a signal, to process signals with finite extent. This way the analysis filter bank produces the same number of samples as in the finite signal, which can then still be completely reconstructed, including the boundary regions.

Example

Figure 1 shows an example of a filter bank with a low system delay, which is switched directly from 1024 to 128 bands. The filters for the steady state case have a length of 4096 and 512 taps resp. and a system delay of 2047 and 255 samples resp., with $\mu = 1$, $\nu = 3$, $n_a = n_s = 1024/2$. Shown are the synthesis baseband impulse responses during the transition.

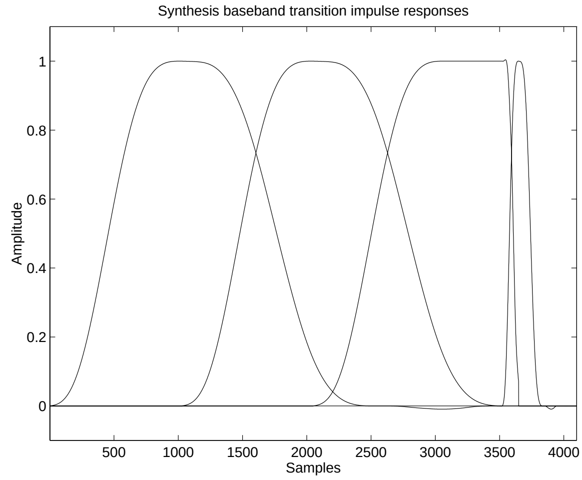

Figure 1. Synthesis transition baseband impulse responses for a switch from 1024 to 128 bands.

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