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A METHOD FOR ALIAS REDUCTION IN CASCADED FILTER BANKS

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ABSTRACT

This paper shows a new way to reduce aliasing in critically sampled cascaded filter bank structures. Unlike standard tree structured methods, which lead to many aliasing components in the final subbands, our approach reduces the effect by canceling aliasing elements among subbands. Our interest lies in compression applications where we can apply the scheme to obtain an unequal or non-uniform band splitting using uniform cosine modulated filter banks. In an example it is shown that a reduction in aliasing of over 40 dB compared to a traditional tree structured filter bank can be achieved.

1. INTRODUCTION

In signal compression applications, often a filter bank with unequal band splitting is desired to obtain maximum coding gain. In audio coding applications, band splitting also determines how well the time and frequency shape of the quantization noise after decoding the signal can be matched to the threshold of human hearing. Here it is desirable to obtain a higher frequency resolution at lower frequencies and a higher temporal resolution at higher frequencies.

To obtain non-uniform band splitting, frequently tree structures of cascaded filter banks are used to implement wavelet transforms [1], where the lower frequency band is further split into subbands. Other approaches include employing a uniform filter bank and joining several subbands into one wider band to increase bandwidth [2]. But this does not increase the temporal resolution sufficiently for many signals. Another approach is combining several pieces of different uniform filter banks by transition filters [3]. But this leads to an increased complexity and to non-perfect reconstruction.

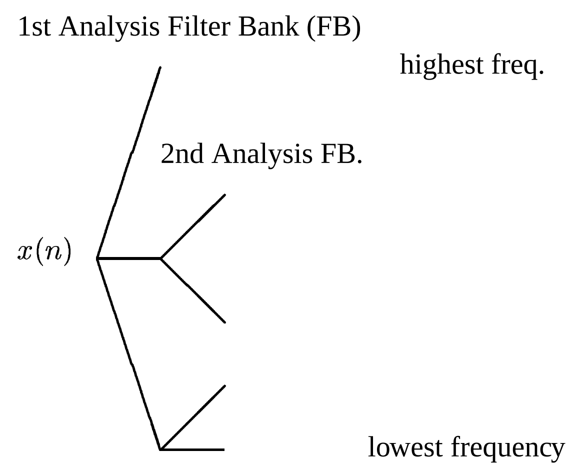

Figure 1: A cascaded filter bank acting on signal $x(n)$. Each branch represents one filter of the filter bank, including downsampling.

A cascaded filter bank has different uniform filter banks following each other to obtain varying frequency and time resolutions, as illustrated in Fig. 1. The problem is that the filters of the first stages often have a limited stopband attenuation and transition bands which span several of the bands at the output of the cascade. Hence the downsampling after each subband filter, which is needed for critical sampling, leads to aliasing. That is, signals with energy outside the passband are "mirrored" into the passband of the

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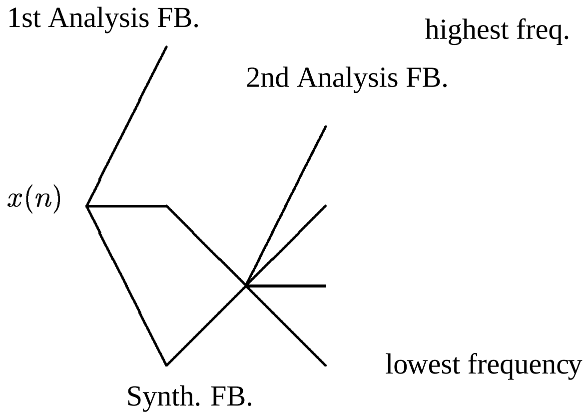

Figure 2: A "stacked" cascaded filter bank, with the same notation as in Fig. 1.

filter. If this mirrored signal is in the passband of the subsequent filter, the attenuation of the signal is only determined by the first filter of the cascade. Although this aliasing does not affect the perfect reconstruction property of the combined filter bank as long as each sub-system is a perfect reconstruction filter bank, it leads to a poor frequency selectivity. This can be seen in Fig. 9, which displays the magnitude response of band 13 of a filter bank as in Fig. 1, consisting of a cascade with 128 bands in the first stage and 8 bands in the following stage. This leads to a total of 1024 bands if all subbands of the first stage are split. The aliasing shows as high peaks in the magnitude response on both sides of the main lobe. As a comparison, the desired magnitude response can be seen in Fig. 8, which shows band 13 of a 1024 band uniform filter bank. An approach to reduce this type of aliasing has been incorporated into the MPEG-1/Layer 3 audio coder [4]. In that coder, a cascaded filter bank is used, with 32 bands in the first stage and 6 or 18 bands (switchable) in the second stage. For this special type of filter bank a butterfly-like structure is used at the output of the second stage to reduce the aliasing between adjacent bands [5]. While this method works for reducing the aliasing in neighboring bands, it only reduces the aliasing from one neighbor, and only in specific filter banks, where the aliasing of neighboring bands has a phase shift of 0 or 180 degrees. In the present paper we extend this idea of alias reduction to act on more than one neighboring band. Our application has the added benefit that it does not require a fixed phase relationship between adjacent bands.

2. NEW SOLUTION

For a given analysis filter bank, the synthesis filter bank obtains perfect reconstruction by using a suitable design to cancel the aliasing components between all subbands. This principle can be used to cancel or reduce the aliasing only between a set of neighboring subbands. To obtain a higher frequency resolution from a set of subbands, the signals entering are first fed into a synthesis filter bank for the reduction of aliasing in this set. Assume the first stage filter bank is a modulated uniform filter bank with M_1 bands and the set consists of M_2 bands, so that the second stage synthesis filter bank for alias reduction has M_2 bands. To obtain alias reduction, the second stage synthesis filter bank has to have the same (or similar) frequency response as the first stage's perfect reconstruction synthesis filter bank, but with the frequency scaled by the ratio of the sampling rates, M_1/M_2. We assume that the uniform filter banks are cosine modulated. That means the second stage synthesis filter bank should have a window function (baseband prototype filter) with a frequency response identical (or similar) to the low frequency portion of the synthesis prototype for the first stage. After the synthesis filter stage used for alias reduction, an analysis filter bank follows to obtain a higher frequency resolution. This is shown in Fig. 2. As an illustration, Fig. 6 shows the low frequency part of the magnitude response of the window function of a 128 band synthesis filter bank. Fig. 7 shows the magnitude response of the 8 band synthesis filter bank used for aliasing reduction. It can be seen that the (scaled) magnitude responses are very similar. The same is also true for the phases (not shown here). This then leads to reduced aliasing.

As long as an analysis filter bank exists which forms a critically sampled perfect reconstruction system together with the synthesis filter stage, the perfect reconstruction property and the critical sampling of the cascaded filter bank are maintained.

3. PROPOSED IMPLEMENTATION

The question is how to obtain a suitable filter bank with a lower number of bands and a similar scaled frequency response, while still achieving overall perfect reconstruction. As described in [6, 7], any cosine modulated filter bank can be represented by a formulation using Zero-Delay and Maximum-Delay matrices. This leads to a composition which consists of nested 2 band sub-structures. Fig. 3 shows an example for a 4 band filter bank, and Fig. 4 the corresponding synthesis filter bank for perfect reconstruction. $\mathbf{T}_1$ is a $M_1 \times M_1$ Discrete Cosine Transform (DCT) type 4 [6], in this case of size 4×4.

Once a first stage filter bank is designed, this structure can be "thinned out" by only keeping every M_1/M_2 th sub-structure. This can be seen in an example in Fig. 5, where a 2 band synthesis filter bank is generated from the 4 band illustration in Fig 4. Here $\mathbf{T}_2$ is a 2×2 DCT. The resulting 2 band filter bank is a perfect reconstruction filter bank by itself, because its structure is invertible. It also has the desired similar frequency response to the 4 band filter bank, but scaled accordingly, and hence can be used for alias reduction.

The values l of the structure determine the resulting window function or baseband prototype of the filter bank. Since this smaller filter bank is obtained by downsampling the structure, its window function is obtained by a similar operation in the first stage filter bank. Since the sub-structures are symmetric around the center of a block of length M_1, the resulting downsampling of the window function appears in an analog fashion, downsampling each interval of length M_1 of the window function symmetric around each interval's center. For example, assume the first stage filter bank has 4 bands ($M_1 = 4$), and its window function has the form

$$1, 2, 3, 4, 4, 3, 2, 1.$$

If the second stage for alias cancellation is to have 2 bands, its window function can be obtained by symmetric downsampling in each block of 4 samples, as follows:

$$2, 3, 3, 2.$$

Since the structural formulation is valid for all cosine modulated filter banks with perfect reconstruction, the downsampling rule is also valid for this entire class of filter banks. That means if a corresponding synthesis filter bank and a window function are given for the first stage analysis filter bank, the window function obtained by the non-uniform symmetric downsampling automatically leads to a perfect reconstruction filter bank. Furthermore, it reduces the aliasing after the first stage.

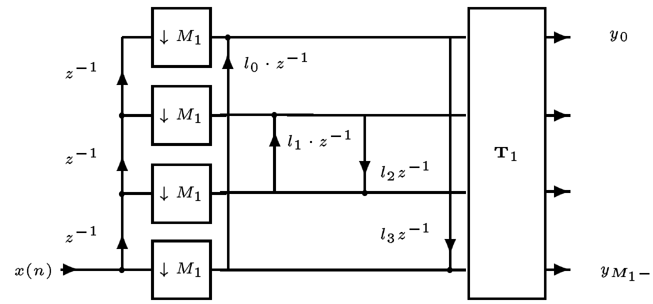

Figure 3: Example of a 4 band analysis filter bank.

This method is illustrated with the following example. The goal of this exercise is to obtain a resolution of 128 (uniform) bands in the first stage and a resolution of 1024 bands in the final stage. Fig. 8 shows an enlargement of band 13 of a uniform 1024 band filter bank, without cascading. With the stacked cascaded filter bank our goal is a frequency response similar to Fig. 8 after the final stage. Fig. 9 shows an enlargement of the same channel but for a traditional cascaded filter bank, with 128 bands in the first stage and 8 bands following in each subband in the next stage. The aliasing is easy to see as high peaks on both sides of the main lobe. To reduce the aliasing, we take a look at Fig. 6. It shows the low frequency portion (the bottom $1/16$ th) of the magnitude response for a baseband prototype of a 128 band synthesis filter bank (perfect reconstruction). The structure of this synthesis filter bank is then downsampled to obtain an 8 band synthesis construction to reduce the aliasing across 8 neighboring bands. Its frequency response can be seen in Fig 7. It can be seen that it is very similar to the low frequency portion of the 128 band filter bank, down to approximately -50 dB. The result leads to a structure of a 128 band analysis filter bank in the first stage, 8 band synthesis filter banks for alias reduction in the second stage, and 64 band analysis in the third (final) stage. The overall synthesis filter bank for this structure contains the reverse steps. The resulting reduction in aliasing can be seen in Fig. 10, which shows the same portion of the spectrum as Fig. 9, but for a stacked cascaded filter bank. It can be seen that the aliasing from cascading is reduced to about -50 dB, which is a reduction by over 40 dB.

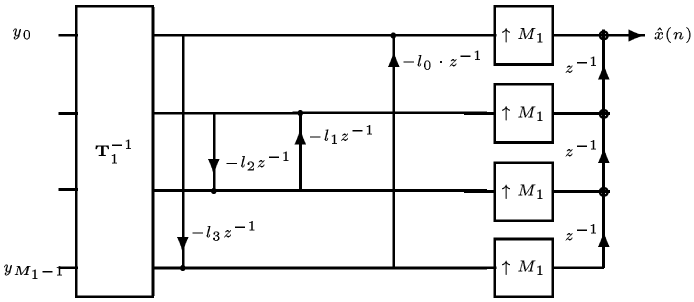

Figure 4: A 4 band synthesis filter bank.

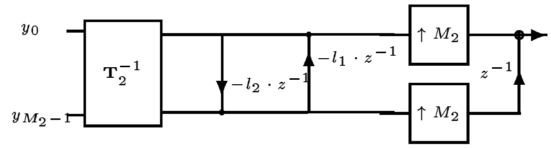

Figure 5: A 2 band synthesis filter bank obtained from the 4 band synthesis filter bank.

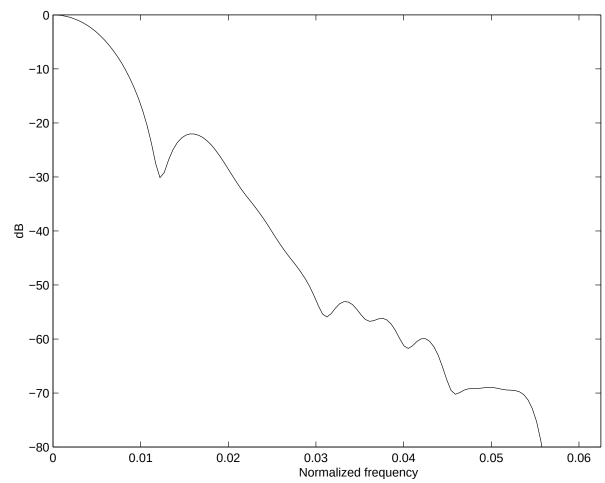

Figure 6: Closeup of the bottom $1/16$ th of the normalized frequency axis of the magnitude response for the baseband prototype of a 128 band synthesis filter bank.

4. CONCLUSIONS

Conventional cascaded filter banks introduce aliasing in the subbands which reduce the frequency resolution and the coding gain in coding applications. Our approach introduces a suitable intermediate stage which reduces aliasing effects, without requiring a fixed phase relationship.

On the analysis side, this intermediate stage consists of small synthesis filter banks which are obtained from the first stage synthesis filter bank for perfect reconstruction. Due to the additional aliasing reduction stage, the proposed technique introduces some additional complexity and delay. However, these factors are outweighed by the advantage that a suitable intermediate stage can reduce the aliasing significantly, in our example by over 40 dB. This way it is possible to obtain a filter bank with a combination of bands with a high frequency resolution and selectivity, and of other bands with the high temporal resolution of the fist stage.

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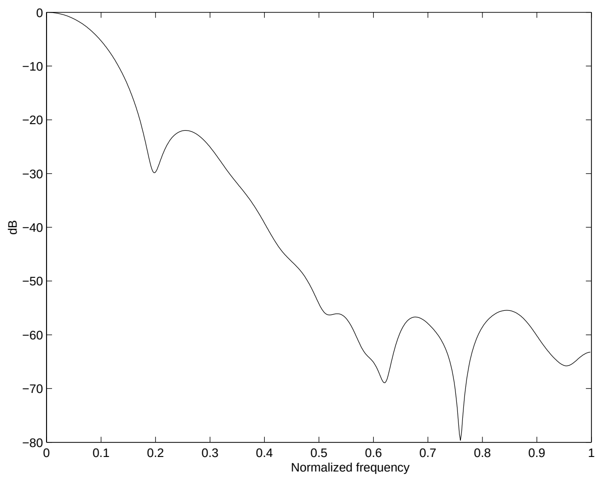

Figure 7: Magnitude response for the baseband prototype of an 8 band synthesis filter bank.

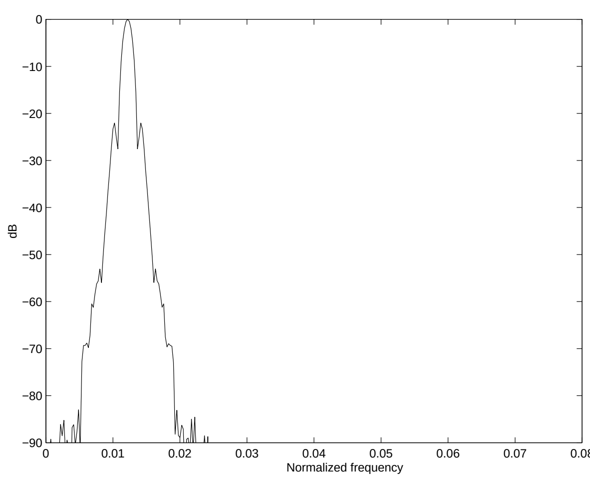

Figure 8: Enlargement of band 13 of the magnitude response of a 1024 band filter bank.

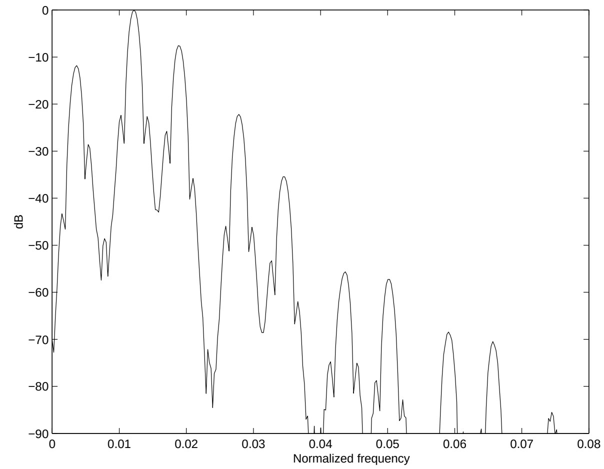

Figure 9: Enlargement of the magnitude response of band 13 of the final stage for a cascaded filter bank with 128*8 bands (128 bands first stage, 8 bands second stage). The aliasing can be seen as high peaks on both sides of the main lobe.

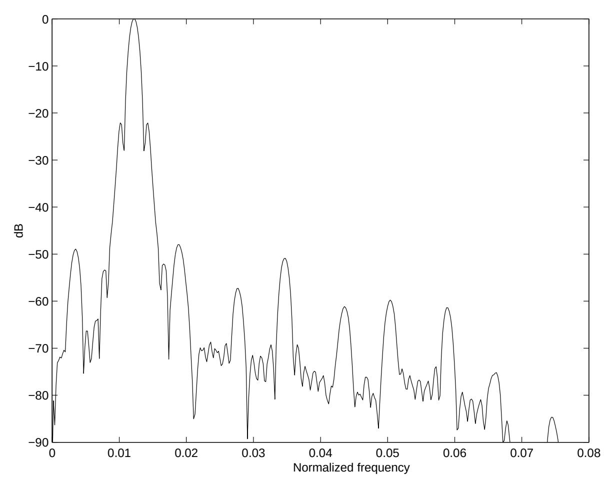

Figure 10: The same enlargement as in Fig. 9, but for a stacked cascaded filter bank with 128/8*64 bands (128 bands first stage, 8 bands synthesis stage, 64 band final analysis stage). It can be seen that the aliasing in the neighboring bands is much reduced.